Using Case Scenarios in Teaching Discrete Mathematics

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July 28, 2017

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- 300-level course with proofs course prerequisite

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- Problem-based guided discovery activities: Explorations and in-class activity pair for each topic

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- Finding an efficient solution method

An Approach: Using Case Scenarios

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If a solution is incorrect, identify why.
If there are multiple correct solutions, explain the validity/equivalency of all correct solutions.
If a solution is incorrect but can be fixed with a minor adjustment, give hints/suggestions to the student to fix their solution.

1. Three students are working on a problem which asks them to count 3-digit strings with at least one of the digits being 1. The first student says

- I got 300. I used the Addition Principle. I define set A to consist of those strings where the first digit is 1 such as 123 or 145. Set B consists of those with the second digit equal to 1, and set C consists of those with the third digit equal to 1. Since at least one digit has to equal 1, $A \cup B \cup C$ gives me the answer. To find the number of elements in A, I found that there are 10 possible options for each of the second and last digits, so there are a total of 100 elements in A. Similarly 100 elements in B and 100 elements in C. Therefore, by Addition Principle
$$|A \cup B \cup C| = |A| + |B| + |C| = 100 + 100 + 100\,,$$
so the answer is 300.

The second student says

- I found 243. I also used the Addition Principle and the union, but I defined my sets differently. My set A consists of those strings where the first digit is 1 but the other digits are not 1. Similarly B includes strings where the middle digit is 1 and the others are not, and C includes those with the last digit equal to 1 and the other digits are not. I found each set to contain 81 elements for a total of 243 elements in all three sets.

The third student says

- I found 271. I first counted all 3-digit strings. Since each digit has 10 options we have 1000 total. Then I counted those strings that do not contain 1. Each digit has 9 options now since 1 is not allowed, so I got 729. So the number of strings we want is 1000-729=271.

Which student's, if any, solution is correct? Explain the student's solution in more detail to justify that it is correct completely. Explain all possible issues with the other students' answers. You can also explain how to fix incorrect answers.

1. Four students are working on a problem together. The problem asks them to find passwords with 6 characters where each character is a lower-case letter or a digit, and the password has at least one digit and one letter.

The first student says

- I found $2 \cdot 10 \cdot 26 \cdot 36^4$. I can place either a digit or letter first, say digit. I get 10 choices for the digit, then 26 for the letter, and the rest of the 4 characters can be anything, which gives me 36^4 and I multiply all of these choices since I choose one after another. If we start with a letter, then a digit, that gives us the same number: $10 \cdot 26 \cdot 36^4$. So my total is $2 \cdot 10 \cdot 26 \cdot 36^4$.

The second student says

- I did something similar to what you did, but I multiplied $10 \cdot 26 \cdot 36^4$ by $\binom{6}{2}$ since the letter and the digit can go into any two spots among the six. They could be in the first two positions, or the last two positions, or the middle two, etc. So I got $15 \cdot 10 \cdot 26 \cdot 36^4$.

The third student says

- I did it differently. I used cases. I considered the five following cases: One letter+five digits; two letter+four digits; three letters+three digits; four letters+two digits; and five letters+one digit. For each case, I decided where the digits and letters would go by choosing spots for them and then multiplied options. For example, for two letters and four digits, I first chose the two spots for my letters, there are $\binom{6}{2}$ ways to do that. Then I placed my letters: 26^2 ways to do that. Then I placed my digits for the remaining spots: 10^4 ways for that. After I added all the cases, I got 1866866560 total.

The fourth student says

- I went the other way around. I first found the passwords we don't want and subtracted that number from all possible 6 character passwords. If we use only letters, we have 26^6 passwords. If we use only digits, we have 10^6 passwords. The number of all possible passwords is 36^6. So I got $36^6 - 26^6 - 10^6$ total.

Which student/s is/are correct? Why are the other students' answers off?

2. The professor who was passing by the group of students in the above problem says "That's awesome. Multiple solution methods. Can you see why that makes sense algebraically? My hint is the binomial theorem." Can you figure out what the professor is referring to?

1. Recall the problem about three students who found the number of 3-digit sequences which have the digit 1 appearing at least once.

After learning about the Inclusion-Exclusion Principle, Student 1 thinks his/her answer can be fixed. Can you help Student 1 to fix his/her answer? Word your solution in such a way that you're writing directly to the student (in a nice and respectful way). Clearly include the problem students were solving and an explanation of the issues within Student 1's solution.

2. Another student who overhears the discussion between the three students above jumps in and says

- But my answer was 268. I used a Venn diagram of the three sets Student 1 defined. In the triple intersection there is one element. In each double intersection there are 10 elements. For example, for the elements in the intersection of sets A and B, the first two digits are 1 leaving 10 options for the last. So that leaves us with $100 - 10 - 10 - 1 = 79$ elements in the portions of sets which are disjoint from the other sets. To find the total we add all the numbers:

$$79 + 79 + 79 + 10 + 10 + 10 + 1 = 268.$$

Which part of this student's thinking is correct, and which part needs fixing?

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- Analyzing solutions and figuring out how to fix mistakes can be frustrating for students.

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- Students gain experience in evaluating others' work and giving feedback.
- Students become more aware of counting mistakes in their solutions and are more careful in evaluating their own work.

Thank you for listening!