The Pythagorean Theorem as a Gateway to Proofs

CPS on Encouraging Effective Teaching Innovation
MathFest, July 28, 2017

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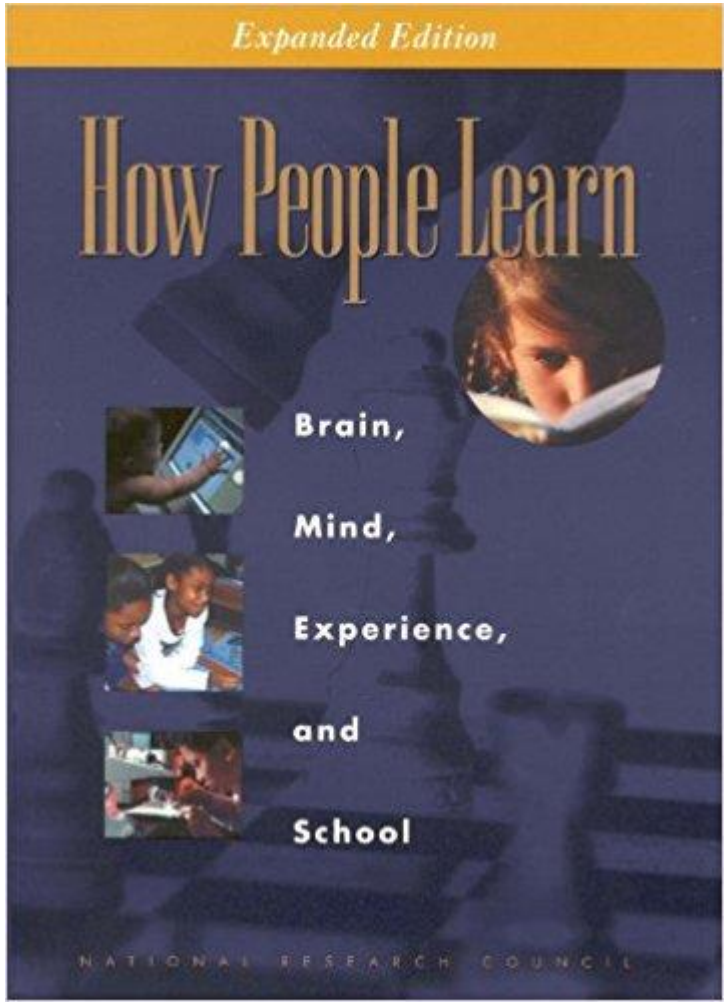
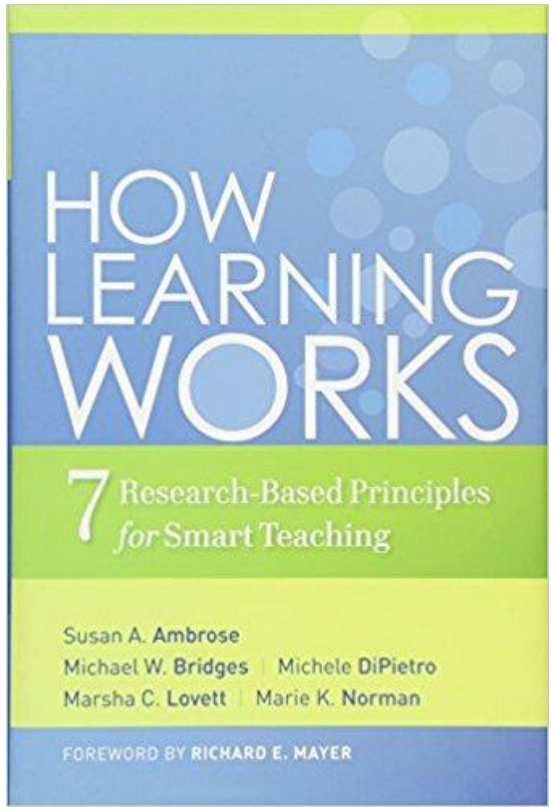
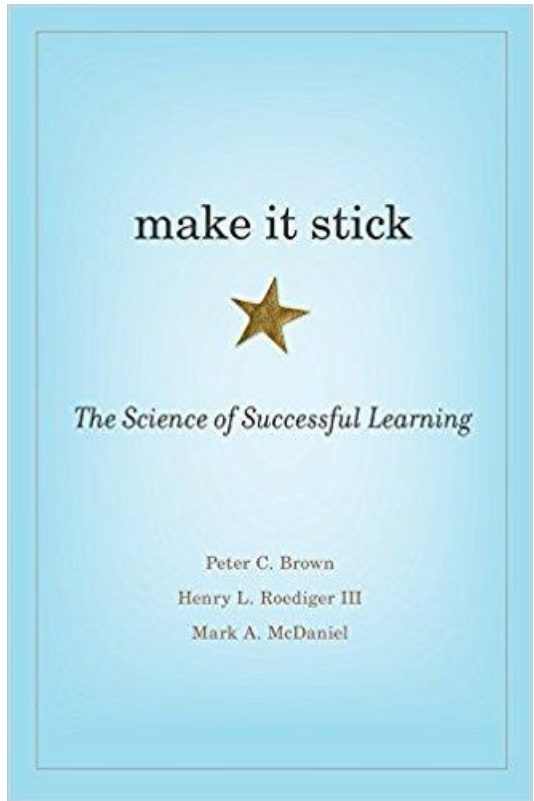
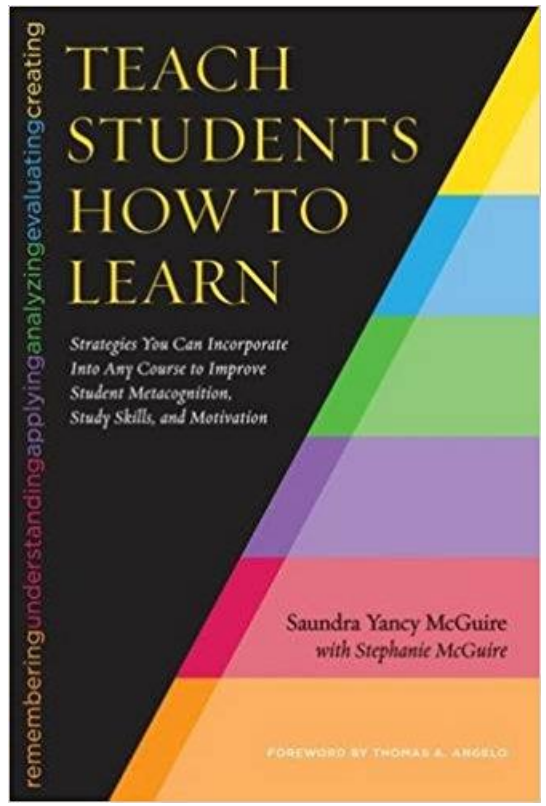


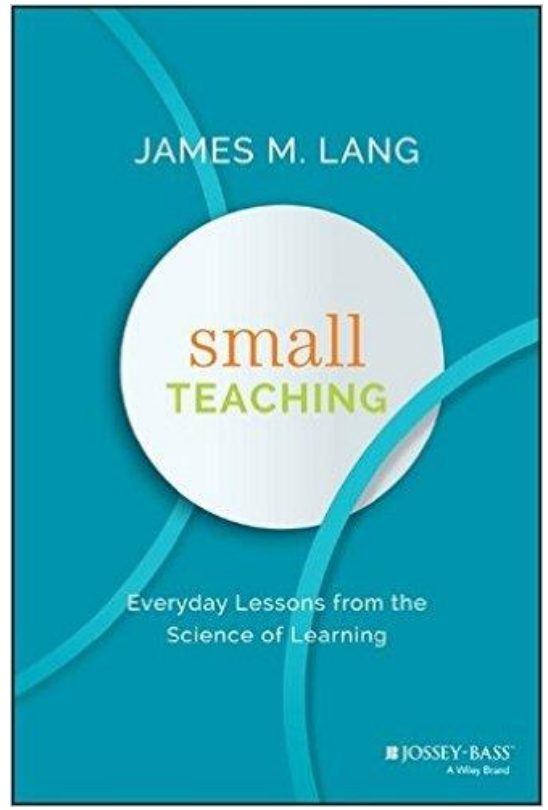
How People Learn
National Research Council, 2000

Available at https://www.nap.edu/catalog/9853/how-people-learn-brain-mind-experience-and-school-expanded-edition


HOW LEARNING WORKS
7 Research-Based Principles for Smart Teaching
Susan A. Ambrose
Michael W. Bridges | Michele DiPietro
Marsha C. Lovett | Marie K. Norman
FOREWORD BY RICHARD E. MAYER


make it stick
The Science of Successful Learning
Peter C. Brown
Henry L. Roediger III
Mark A. McDaniel


remembering understanding applying analyzing evaluating creating
TEACH STUDENTS HOW TO LEARN
Strategies You Can Incorporate Into Any Course to Improve Student Metacognition, Study Skills, and Motivation
Saundra Yancy McGuire
with Stephanie McGuire
FOREWORD BY THOMAS A. ANGELO


JAMES M. LANG
small TEACHING
Everyday Lessons from the Science of Learning
JOSSEY-BASS
A Wiley Brand

How People Learn: Takeaways

- Experts see problems and content differently than novices do
- Transfer of knowledge is hard
- New knowledge is built on old (and sometimes old knowledge must be cleared away)
- Skills mastery helps support deeper advanced learning

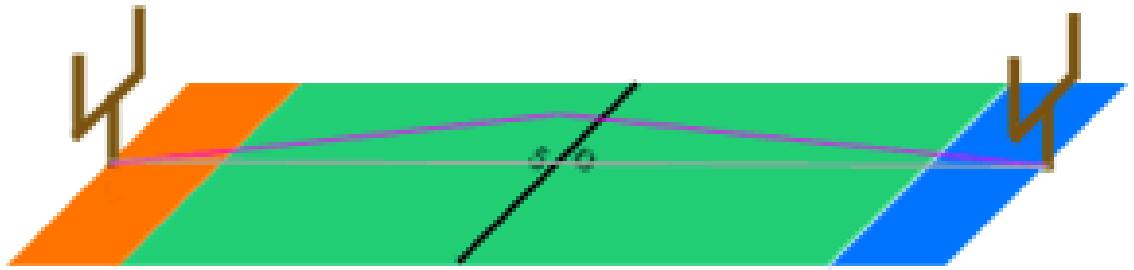
Pythagorean Theorem Unit – Design Goals

- Activate existing knowledge (statement of the Pythagorean Theorem)
- Discuss preconceptions/misconceptions about proofs
- Use good pedagogy: active learning, collaboration, students presenting to peers, writing, etc.
- Introduce new ideas and concepts by inspiring a “need to know”

Think-Pair-Share: Proofs

1. In your own words, what is a mathematical proof?
2. Do you have a positive or negative association with proof? Support your feelings with a few examples of your experiences so far with mathematical proof.

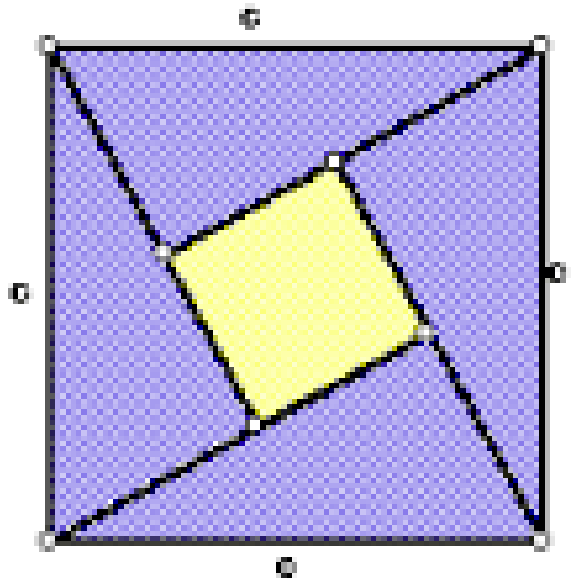
How high is the rope?

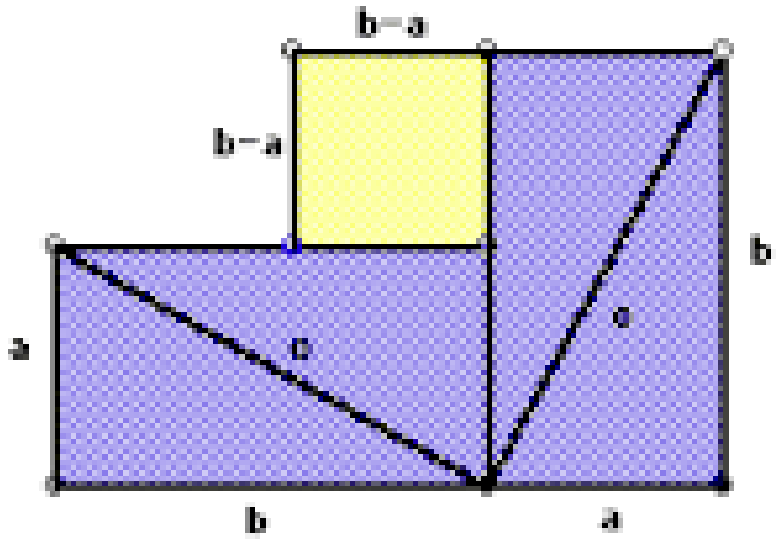


A. Not high enough to fit my finger under it
B. Just high enough to crawl under
C. Just high enough to walk under
D. High enough to drive a truck under

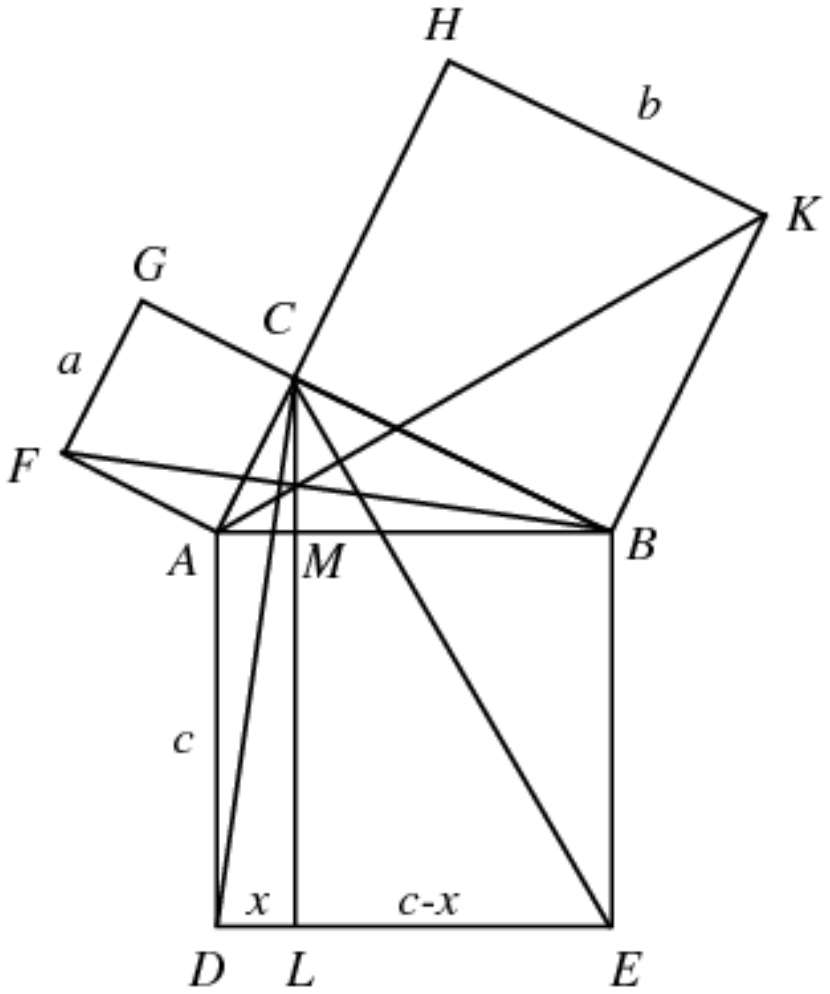
Su, Francis E., et al. "Football Field." *Math Fun Facts*. <http://www.math.hmc.edu/funfacts>

Our First Proof: Bhaskara's "Puzzle"



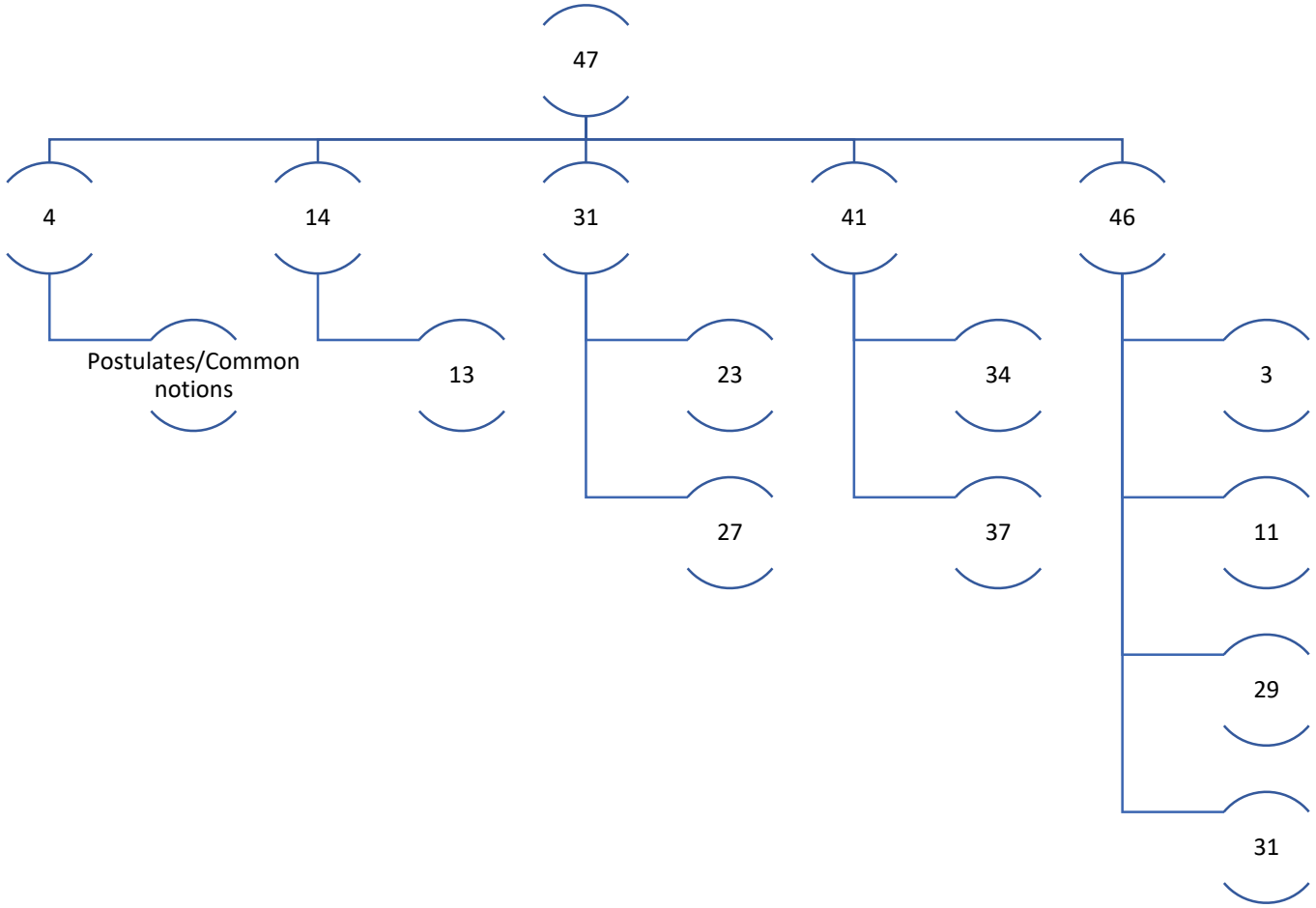


The “Bride’s Chair”



Schopenhauer: a “brilliant piece of perversity”

Euclid's Proposition 47: The Pythagorean Theorem



Later Connections

The various roles of proof:

- Michael de Villiers, “The Role and Function of Proof in Mathematics,” *Pythagoras,* November 1990

More “famous” proofs, including:

- There are infinitely many prime numbers (using Euclid’s own proof).
- The square root of 2 is irrational.
- The infinity of the set of real numbers is larger than the infinity of the natural numbers.