

Visualizing Mathematical Reasoning: A Diagrammatic Approach

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Presentation Outline

Introducing Proof Maps

Pedagogical Uses

Classroom Experience

Next steps

Motivation

Clarify the structure of proofs by breaking it into discrete logical steps using diagrams.

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- ▶ Contradiction

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- ▶ Logical Flow

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- ▶ Cases
- ▶ Contradiction
- ▶ Logical Flow
- ▶ What is being proved?

Searching for structure

Much of the work of working through a proof is determining its structure

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Stripping away the language helps this process

Arguments are typically non-linear

- ▶ Diagrams are appropriate

Definition

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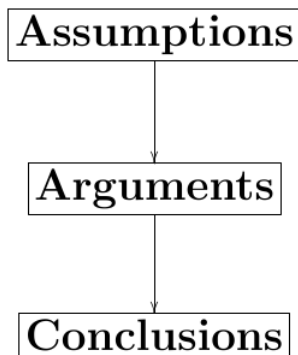
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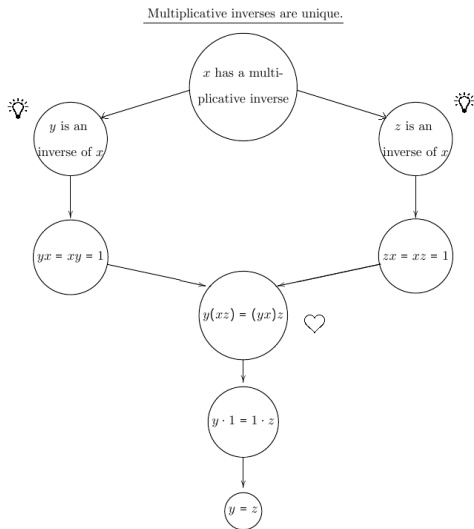
A proof map is a diagrammatic representation of a proof

- ▶ Main steps of the proof are written in bubbles
- ▶ Lemmas used are written in boxes
- ▶ Arrows are drawn in between to denote implication

Basic Format



Basic example



Additional Notation

Additional notation to accelerate proof interpretation

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- ▶ Hearts next to the most important items

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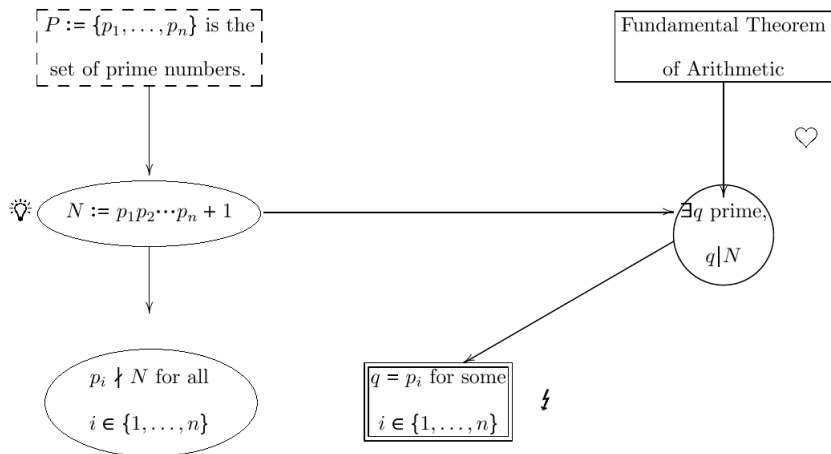
Additional Notation

Additional notation to accelerate proof interpretation

- ▶ Hearts next to the most important items
- ▶ Lightbulbs next to the clever tricks
- ▶ Dotted bubbles around assumptions made towards a contradiction
- ▶ Lightning bolts next to the contradictions

Another example

Proof by Contradiction



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Less language, more comprehension

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- ▶ Subtle linguistic choices convey messages which the students may miss
- ▶ Easier to remember
- ▶ Trains students to analyze proofs the way mathematicians do

Reinforcing visual learning

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- ▶ Makes logical gaps more obvious

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Implemented proof map techniques in an introduction to math reasoning course

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- ▶ Workshop 30 minutes a week in addition to homework problems

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Messy data

- ▶ Unable to deduce concrete results

Helpful techniques

Proof maps may be useful in creating a workshop for math reasoning courses

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- ▶ Matching tended to be “meta-reasoned”

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- ▶ Fill-in-the-blank and connect-the-bubbles spurred engagement
- ▶ Matching tended to be “meta-reasoned”
- ▶ Proof mapping a part of a proof may be appropriate early on

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Going forward

Experimenting with a slightly higher level course

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- ▶ Considering introducing them into an into to analysis course

Thank you!

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