"I haven't had to analyze my own thinking very much before,
at least not in math. I feel like I'm saying that a lot –
this math class is different from all the others. It's true though,
because I haven't had to think about the way I'm thinking.
It's just, I thought about it, and it's done.
This class has helped me with analyzing that."

Using Portfolio Problems to Develop Metacognitive Thinking During Problem Solving

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UNIVERSITY OF NORTHERN COLORADO

Motivation: "Authentic" Problem Solving

Problem-Solving Metacognitive Actions (Based on Carlson & Bloom, 2005)	
MA 1.	Mathematical concepts, knowledge, tools, and facts are assessed and considered
MA 2.	Various solution approaches or strategies are assessed and considered
MA 3.	Solution process is assessed and considered for validity/reasonableness
MA 4.	Results are assessed and considered for validity/reasonableness
MA 5.	Reflects on the efficiency and effectiveness of cognitive activities
MA 6.	Manages emotional responses to problem-solving situation

Product → Process

Think Mathematically

Habits of Mind

Motivation: Rethinking Intervention

"Most efforts to directly teach metacognitive skills and other deliberate learning strategies have been disappointing.

***The taught skills often are not retained, are not applied independently by students, or take a brittle form** that does not seem to enhance other learning, even when the new strategies themselves are performed to specification.*

*A repeated finding is that **general strategies directly taught to students tend not to be spontaneously used under conditions different from those in which they were initially practiced**."*

(Greeno, Collins, and Resnick , 1996, *p. 35)*

Motivation: Rethinking Intervention

Metacognitive instruction should be **embedded in mathematics content** and take place for an **extended period of time.**

(Lester, Garofalo, & Kroll, 1989; Veenman, Van Hout-Wolters, Afflerbach, 2006)

The Class

- First-year, first-semester course for **pre-service elementary education majors** with mathematics emphasis

- **Instructional Team:** Instructor of Record ('Dr. G') + participant/researcher

- **Inquiry-oriented instruction as foundation**
  - "Student engagement in rich mathematical tasks"
  - Group-work centered with "regular opportunities for student-to-student and student-to-instructor collaboration"
  - Inquiry-Based Learning (IBL) classroom practices allowed us to:
    - **Provide *all students* opportunities to participate in the *process* of problem solving**
    - Negotiate common problem-solving language

(Academy of Inquiry Based Learning)

The Portfolio Problems

Worthwhile-Problem Criteria (NCTM, 2010):

The problem has important, useful mathematics embedded in it.

The problem requires higher-level thinking and problem solving.

The problem can be approached in multiple ways using different solution strategies.

The problem has various solutions or allows different decisions or positions to be taken and defended.

The problem encourages student engagement and discourse.

Portfolio Problem 5 [Unit 6: Ratio and Proportion, Functions]

At sunrise two old women started to walk towards each other. One started from point A and went towards point B while the other started at B and went towards A. They met at noon but did not stop; each one continued to walk maintaining her speed and direction. The first woman came to the point B at 4:00 pm, and the other one came to point A at 9:00pm. At what time did the sun rise that day?

In-Class Sessions

Logistically consistent with overall group-work centered course design

Each group member wrote in a different colored pen on scratch work to identify individual contributions

Group scratch work emailed to group after class

Some instructor feedback provided between sessions

		Tuesday	Thursday
Unit 1 Counting, Natural Numbers, Place Value	Week 1		PPS 1
Unit 2 Meaning/Interpretation of Arithmetic Operations $+, -, \times, \div$	Week 2		PPS 2
	Week 3	PPS 2	
	Week 4		PPS 3
Unit 3 Factors, Multiples, Prime Factorization, GCF, LCM	Week 5		PPS 3
	Week 6		
Unit 4 Meaning/Representation of Fractions, Decimals	Week 7		
	Week 8		
	Week 9		PPS 4
Unit 5 Expressions, Equations, Solving Equations	Week 10	PPS 4	
	Week 11		
Unit 6 Ratio and Proportion, Functions	Week 12		PPS 5
	Week 13	PPS 5	
	Week 14	PPS 6	PPS 6
	Week 15		

Written Portfolio Submissions

Portfolio Problem Set 3

Instructions:

(1) Submit your Portfolio Problem Set 3 together with Homework 5 using the ASSIGNMENT 5 cover and the checklist page.

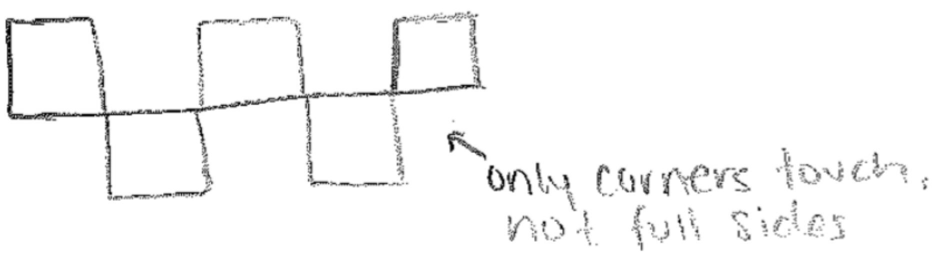
(2) Note that the problem on Portfolio Problem Set is much more involved than the ones on homework, and the intention is to allow freedom to roam with it in any direction you wish.

The portfolio problem submission will consist of the following:

- **Scratch Work:** Submitting every piece of writing related to your work on this problem. This includes scrap work of ideas, any attempts even if scratched out (preferably not erased), all polished solutions or special cases, all computations, and all recordings of joint work (i.e., your work in group discussion from last week Thursday's class and outside work with group members).
- **Revised Solution (at least 3 pages):** You are asked to work on the problem further. This may mean you can investigate one of the solution paths you started in class with group members, etc. You need to provide at least one revised solution, neatly written so that a reader can follow your reasoning and ideas. In addition to the solution, include a write-up to address the following items:
    - CLEAR Explanation of what you revised from scratch or why choose to do a new exploration (why did you not use the ideas in scratch work).
    - Explanation of your thinking and reasoning about your revisions.

Written Portfolio Submissions

After getting this portfolio problem, I didn't really know where to start. First, I decided to figure out what a pentomino was. My group and I wondered if the shape below was a pentomino.



After reading the explanation, again, we came to the conclusion the "w" above would not be considered a pentomino because they do not touch another square on a full side. Instead, they connect at the corners.

Next, we started testing different numbers to see if there was a certain pattern. I began with the letter "p" in the top left corner and found that the five numbers it covered (1, 2, 11, 12,

Written Portfolio Submissions

I asked myself "if the shape is made of 5 squares, then why wouldn't it be divisible by 5?" The answer to the questions lies within the shape of the pentomino and the numbers it is placed over. If the chart had the same number in every box, instead of increasing numbers, than the number would be divisible by 5 every time.

Example:

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

*No matter where you put the letter, the answer will always be divisible by 5 because there are 5 groups

1	2	3	4	5
11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45

* Because the one's place changes in each column, the numbers may not be divisible by 5 even though there are 5 squares.

Grading Portfolio Submissions

Portfolio problem submissions were worth 15% of students' overall course grade

Each submission was graded on a 15-point scale:

Scratch Work (3 points)
Graded for completion

3-page revised solution
"Complete Solution" (4 points)
Mathematical reasoning, justification and all related computations
Explanation of Revised Solution (4 points)
Which solution path was used and why
Why ideas from scratch work were implemented or not implemented
Other observations made about your thinking and problem solving process

Accuracy of mathematical work (4 points)
Accuracy of arithmetic calculations, use of mathematical notation, use of mathematical properties, etc.
Accuracy ≠ Finding "THE ANSWER"

Results: Retroactive Product to Proactive Process

General Classroom Environment:

"It's all about the asking of questions that made this course successful…**I've never really thought of to ask myself 'Why did you do this?'** I've never really done that before."

"Reflecting upon what you did, that was definitely emphasized in this course. You were supposed to **reflect on even your very first thoughts** when you see a problem."

Results: Retroactive Product to Proactive Process

In-Class Portfolio Problem-Solving Sessions

"I remember working on the first portfolio problem and trying to explain, 'This is what I'm thinking, and this is why.' My entire group was just looking at me like 'What are you talking about? I don't understand what you're trying to say.' Then as the year goes on, **I can look back and see myself getting better at talking about what I'm thinking.**"

Results: Retroactive Product to Proactive Process

Scratch Work

"**I think also writing [questions] down makes them more important**…I feel like writing it down makes me realize like, 'Oh I probably had a reason that I wanted to answer it, because it would get me to another step or get me to understand something else.'"

Results: Retroactive Product to Proactive Process

Written Submissions

“I've just been able to be **actively engaged in the problem, realizing what I'm doing**. Rather than just like, ‘Well, this is the first step and second step,’ and then afterwards I'm like, ‘Oh, that was wrong, and that was wrong.’"

“You're writing it and…it's either ‘Why does this make sense?’ or like ‘How is that correct?’ Because sometimes **I remember writing it and I'm like, ‘Wait, this doesn't make sense,’…**

Some Future Considerations

- Student buy-in
  - Clear understanding of course goals
- Student collaboration
  - Students often wrote on their own paper
- Grading scratch work
  - Holds students accountable
  - Not currently used as formative assessment

Make it better and let us know!

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Visit me at my poster during the IBL mini-conference

Questions?

THANK YOU!

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